SIKKIM UNIVERSITY

M. Phil/Ph.D. Course Work Syllabus

Course Code	Course Name	Credit
MAT-RS-C501	**Research Methodology**	**4**
MAT-RS-C502	**Research Proposal and Preparation**	**4**
MAT-RS-E503	**Differential Quadrature method and Applications**	**4**
MAT-RS-E504	**Mesh-free methods and Applications**	**4**
MAT-RS-E505	**Computational method for PDE**	**4**
MAT-RS-E506	**Commutative Rings**	**4**
MAT-RS-E507	**Value Distribution Theory**	**4**
MAT-RS-E508	**Advanced Linear Algebra**	**4**
MAT-RS-E509	**Advanced Functional Analysis**	**4**
MAT-RS-E510	**Numerical Linear Algebra**	**4**
MAT-RS-E511	**Homological Algebra**	**4**

Total credits=12

Abbreviations: C-Core(Compulsory), E-Elective

RESEARCH METHODOLOGY
MAT-RS-C501

3-1-0-4

MAT-RS-C501: Research Methodology and Advanced Mathematics

UNIT I: Used of computer/Programming LaTex and Beamer typing Concept of computer language-Fortran 90 programming, Matlab, Mathematica, CoCoa software, Origin

UNIT II, III &IV : Quantitative method & Problem solving in Mathematics Problem solving in Algebra and Number Theory, Problem Solving in Analysis and Differential Equations, Problem Solving in Topology and Complex Function Theory.

RESEARCH PROPOSAL AND PREPARATION
MAT-RS-C502

Unit I & II: Review of a scientific research paper
Studying a research paper and writing a review of the same, identifying any new problem, question, and direction
emanating from the paper.

Unit III & IV: Research proposal
The students will write a detailed proposal of their research including a thorough review of literature on a topic of
their choice and present the same in a seminar at least 10 days before the End-Semester examination.

DIFFERENTIAL QUADRATURE METHODS AND ITS APPLICATIONS
MAT-RS-E503

Unit-I: Introduction to Differential Quadrature
Introduction, differential quadrature, analysis of linear vector space, properties of linear vector space, solutions of
partial differential equations and function approximations, Fourier series expansion, general functions, even functions,
odd functions.

Unit-II: Polynomial based differential quadrature (PDQ)
Computation of weighting coefficients of the first order derivative, Bellman's approaches, Quan and Chang's approach,
Shu's general approach.Computation of weighting coefficients for the second and higher order derivatives, Shu's
recurrence formulation for higher order derivatives.Matrix multiplication approach.

Unit-III: Fourier expansion based differential quadrature (FDQ)
Cosine expansion based differential quadrature (CDQ) for even functions, Sine expansion based differential quadrature
(SDQ) for odd functions, Fourier expansion based differential quadrature (FDQ) for general functions.

Unit-IV: Solution techniques for differential quadrature (DQ) resultant equations
Solution techniques for differential quadrature of ordinary differential equations (ODEs), Implementations of boundary conditions.Sample applications of DQ method to Burgers' equation, Two-dimensional Poisson equation and Helmholtz eigenvalue problems.

Text Book:
1. C. Shu, Differential Quadrature and its application in Engineering, Springer-Verlag London Ltd., Great
Britain, 2000.
2. ZhiZong and Yingyan Zhang, Advanced Differential Quadrature Methods, CRC press, London, 2009.

MESH-FREE METHODS AND APPLICATIONS
MAT-RS-E504

Unit I: Overview of meshfree methods
Why Meshfree methods, Definition of Meshfree methods, Solution procedure of MFree methods, Categories of
Meshfree methods, Classification according to the formulation procedures-Meshfree methods based on weak-forms-
Meshfree methods based on collocation techniques-Meshfree methods based on the combination of weakformand
collocation techniques.

Unit II: Meshfree methods based on the moving least squaresapproximation
Moving least squares shape functions, Formulation of MLS shape functions, Choice of the weight function, Properties
of MLS shape functions, Examples of MLS shape functions, Interpolation error using Meshfree shape functions,
Fitting of a planar surface, Fitting of a complicated surface.

Unit III: Element Free Galerkin Method
EFG Formulation with Lagrange Multipliers, EFG with Penalty Method, Some simple applications.

Unit IV: Meshless Local Petrov–Galerkin Method
MLPG Formulation, The Idea of MLPG, Formulation of MLPG, Types of Domains, Application to some simple
problems**.**

Text Books:
1. G.R. LIU, Y.T. GU, AN INTRODUCTION TO MESHFREE METHODS AND THEIR PROGRAMMING,
Springer -2005.
2. G.R. LIU, MeshFree methods: moving beyond finite element methods, CRC Press London, 2003.

COMPUTATIONAL METHODS FOR THE PDE
MAT-RS-E505

Unit – I: Partial Differential Equations
Introduction, Difference methods, Routh Hurwitz Criterion, Domain of dependence of hyperbolic equations

Uni t – II: Difference methods in Parabolic PDEs
Introduction, One space dimension, Two space dimensions, variable coefficients problems, spherical and cylindrical
coordinate systems

Unit –III: Difference methods for hyperbolic PDEs
Introduction, One space dimension, Two space dimensions, first order equations, systems of first order equations

Unit – IV: Numerical methods for elliptic PDEs
Difference methods for linear BVPs, General second order linear equations, quasilinear elliptic equations

Text Book:

1. Williams F Ames, Numerical Methods in PDE, Academic Press, New York, 1977.
2. Paul Duchateau and David W Zachmann, Partial Differential Equations – Schaum's Outline Series, McGraw-
Hill, 1986

COMMUTATIVE RINGS
MAT-RS-E506

UNIT I: Prime Ideals and applications
Prime ideals, G-domains, G-ideals, Hilbert rings, Hilbert Nullstellensatz

UNIT II: Localization and Integral Extension
Localization, Prime ideals in polynomial rings,Integralextensions,Going-up and Going-down theorems, Valuation
domains, Prufer domains and Bezout domains.

UNIT III: Noetherian Rings & Factorization
Noetherian rings, Hilbert basis theorem, Krull's intersection theorem,
Nakayama lemma, Zero divisors, Discrete valuation rings, Dedekind domains, Krull domains.

UNIT IV: Cohen Macaulay & Regular Rings
R-sequences, Cohen-Macaulay rings, Principal ideal theorem, Generalised principal ideal theorem, Regular rings.

Text book:
1. Commutative Rings by Irving Kaplansky, Chicago universitypress,1968.

Reference Books:
1. Commutative Ring Theory by Hideyuki Matsumura, Cambridge studies in advanced mathematics 8, Cambridge
universitypress,Cambridge,1989.
2. Introduction to Commutative Algebra by M.F.AtiyahandI.G. Macdonald, Addison- Wesley Publ. Company,
1969.
3. Local Algebra by Jean-Pierre Serre(translated from theFrench by CheeWhye Chin), Springer,1999.

VALUE DISTRIBUTION THEORY
MAT-RS-E507

Unit – I: Review of general theory of entire and meromorphic functions
Harmonic functions and their relations with analytic functions, Poisson-Jensen's formula, Elliptic function, Nevanlinna's
characteristic function and related results.
Unit – II: Growth properties of entire and meromorphic functions
Growth indicators of functions, order, hyper-order, basic properties and related results.

Unit – III: Deficiencies of meromorphic functions and their generalizations
Various types of deficiencies of different functions at some given point, their inter-relations and relevant results.

Unit – IV: Uniqueness of entire and meromorphic functions sharing values

Basic uniqueness theorems on analytic functions and their counterparts for meromorphic functions, value sharing,
uniqueness results under value sharing.

Texts/References Books:
1. W. K. Hayman: Meromorphic Functions, The Clarendon Press, Oxford, (1964).
2. C. C. Yang and H. X. Yi: Uniqueness Theory of Meromorphic Functions, Science Press, Beijing (2003).
3. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).
4. L. Yang, Value Distribution Theory.
5. A. S. B. Holland: Introduction to the theory of Entire Functions, Academic Press, New York (1973).

ADVANCED LINEAR ALGEBRA
MAT-RS-E508

Unit I: Eigenvalues, Eigenvectors and Canonical forms
Eigenvalues and eigenvectors, diagonalization, invariant subspaces and triangularization, minimal polynomial, Jordan canonical form with applications, rational canonical form.

Unit II: Inner Product Spaces
Hermitian, normal and unitary matrices, Schur's theorem - real and complex versions. Spectral theorems for normal
andHermitian matrices - real and complex versions. Positive definite matrices, characterizations of definiteness.
Congruence and simultaneous diagonalization.Singular value decomposition, polar decomposition.

Unit III: Hermitian and Symmetric Matrices
Variational characterizations of eigenvalues of Hermitian matrices, Rayleigh-Ritz theorem, Courant-Fischer theorem, Weyl theorem, Cauchy interlacing theorem, Inertia and congruence, Sylvester's law of inertia.

Unit IV: Localization and perturbation of eigenvalues
Matrix norms, spectral radius formula, relationships between matrix norms.Gerschgorin discs, perturbation theorems
and other inclusion regions. Functions of matrices via spectral decompositions.

Texts/References:
1. R. A. Horn and C. R. Johnson, Matrix Analysis, CUP, 1985.
2. S. Axler, Linear Algebra Done Right, 2nd Edition, UTM, Springer, Indian Edition, 2010.
3. P. Lancaster and M. Tismenetsky, The Theory of Matrices, Second edition, Academic Press, 1985.
4. F. R. Gantmacher, The Theory of Matrices, Vol-I, Chelsea, 1959.

ADVANCED FUNCTIONALANALYSIS
MAT-RS-E509

Unit I: Bounded linear operators on Banach spaces
Banach spaces, bounded linear operators, open mapping theorem, closed graph theorem, uniform boundedness principle.

Unit II: Weak and weak* topologies
Dual spaces, Hahn-Banach theorem, transpose of a bounded linear operator, weak and weak* topologies, Alaoglu
theorem.

Unit III: Bounded operators on Hilbert spaces
Hilbert spaces, orthonormal bases, Riesz representation theorem. Adjoint of a bounded linear operator, orthogonal
projection, projection theorem, self-adjoint, normal and unitary operators.

Unit IV: Spectral Theory
Spectrum of a bounded linear operator, Gelfand-Mazur Theorem, Compact operators, Riesz theory for compact
operators, spectral theory of compact self-adjoint/normal operators.

Texts/References:
1. J. B. Conway: A Course in Functional Analysis, 2nd edition (Springer low price edition)
2. B. V. Limaye: Functional Analysis, 3rd edition (New Age Publishers)
3. G.F. Simmons: Introduction to Topology and Modern Analysis (McGraw Hill Education)

NUMERICAL LINEAR ALGEBRA
MAT-RS-E510

Unit I: Linear systems
LU decompositions, Gaussian elimination with partial pivoting, banded systems, positive definite systems, Cholesky
decomposition.

Unit II: Sensitivity of Linear Systems
Floating point computations, IEEE floating point arithmetic, analysis of roundoff errors; Sensitivity analysis linear
and condition numbers, sensitivity analysis of linear systems, stability of Gaussian elimination..

Unit III: The Least Squares Problem
Householder transformation, Givens rotations; QR factorization, stability of QR factorization. Solution of linear least squares problems, normal equations, singular value decomposition(SVD), Moore-Penrose
inverse. Sensitivity analysis of least-squares problems.

Unit IV: Eigenvalue Problems
Eigenvalues, eigenvectors, Schur decomposition, reduction to Hessenberg and tridiagonal forms. Power, inverse
power and Rayleigh quotient iterations. Explicit and implicit QR algorithms for symmetric and nonsymmetric matrices,
sensitivity analysis of eigenvalues. Reduction to bidiagonal form, Golub- Kahan algorithm for computing SVD.

Texts/ References:
1. D. S. Watkins, Fundamentals of Matrix Computations, 2nd Ed., John Wiley, 2002.
2. L. N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM, 1997.
3. G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd Ed., John Hopkins University Press, 1996.

HOMOLOGICAL ALGEBRA
MAT-RS-E511

Unit 1: Review of Rings and Modules, categories and functors**,** Additive and abelian categories,

Unit 2: Satellites, Exact sequences, Projective and injective objects, Homology

Unit 3: Derived functors, Applications: Tor and Ext; group cohomology; sheaf cohomology

Unit 4: Spectral sequences, Applications of Spectral Sequences, The derived category

Recommended Books:

1. Homological Algebra - by H. Cartan, S. Eilenberg.
2. An introduction to homological algebra - C. A. Weibel.
3. A user's guide to spectral sequences- J. Mc Cleary.
4. An introduction to homological algebra - J. J. Rotman.